



# (Towards) A prototype stochastic general equilibrium model of the global food system

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MACSUR Mid-term meeting, Sassari, April 1-4, 2014





# Outline

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- ▶ A simple case treated with different models
- ▶ Conclusions thereof





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- ▶ They face risks of ill-being (bankruptcy / food shortage) because of this — presumably growing larger with climatic change
- ▶ We analysts are uncertain with respect to how much the risks grow
- ▶ Unless our uncertainties are modeled and quantified in appropriate ways joint with those of producers and consumers, we will *almost certainly* give wrong advice



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- ▶ Basically, the assumption on insurance is false. Making insurance markets work is costly. An infinite number of such markets are required
- ▶ An alternative (Radner 1972) is to specify a few insurance markets: general equilibrium with incomplete asset markets (GEI)





## A case

- ▶ A tribe farms by making efforts,  $x$ , on their land and reaping their crops,  $y$ , according to what nature,  $w$ , allows. The crop model,  $y = F(x, w)$ , is known and invariant, but the outcome of nature is known only by its invariant probability distribution,  $\pi(w)$ .





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- ▶ A utility function,  $U(x, y)$ , measures the well-being of the tribe in a single year
- ▶ The chief decides a strategy with respect to effort,  $x^*$ , depending on the model he leans on, static GE or stochastic dynamic GEI







## Making the case more realistic

- ▶ The tribe may find ways to store crops from one year to the next: Barns and/or livestock
- ▶ The tribe may trade with other tribes forming markets at certain points in time
- ▶ Tribes may specialize in wage labor or production management according to endowments and resources
- ▶ Markets for insurance contracts can be formed to spread risk
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- ▶ *but none of these modifications change the fundamental contrasts between GE and GEI*





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- ▶ The marginal effect of nature on well-being is given by derivative of the value function (envelope theorem):

$$\partial_w U^*(w) = \partial_y U(x^*, F(x^*, w)) \partial_w F(x^*, w)$$

The dependence of  $x^*$  on  $w$  plays no role for the marginal effect





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$$U^*(y, \pi) = \max_x \{ U(x, y) + \beta E_{\pi(w)} U^*(F(x, w), \pi) \}$$

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- ▶ The marginal effect of *a change in the probability distribution of nature* on well-being can be calculated

$$\partial_{\pi} U^*(y, \pi) = \beta \partial_{\pi} E_{\pi(w)} U^*(F(x, w), \pi)$$





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- ▶ Ensemble prediction of nature, ensemble crop model and ensemble utility function play no role alone





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  - ▶ more difficult than GE models. Differential equations need be solved — approximately
  - ▶ the big picture can presumably be painted with relatively low resolutions over within-year time, between-year time, space, commodities and states of nature



Thanks for your attention



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