The problem of series of days without rainfall in a view of efficiency of agricultural output under climate change

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UTP University of Science and Technology in Bydgoszcz, Poland is a multi-profile school of higher education offering technical, agricultural, art and in field of management study programs.

Modelling future is key issue in studying CC impacts on agriculture across disciplines and scales. Improving models based on empirical data coming from diverse micro regions facilitate synergic effects important in promoting food security. Rainfall distribution is one of the most important factors determining agricultural output.

Statistical analysis were done for the period from 1st January 1971 to 31st December 2015. The purpose of the analysis was to examine two characteristics: the amount of precipitation per unit area and the length of the series of days without precipitation.

The values of the basic statistics are as follows:

For precipitation:

- Number of observation days N = 16434
- Number of days without precipitation n = 10886
- Average value = 3.874
- Standard deviation = 5.423
- Min = 0.1
- Max = 84.6
- Median = 2.2

Parameters of gamma distribution determined by the most reliable method:

α = 0.659 β = 5.545

By the tests of λ-Kolmogorov and Pearson χ2 it is stated that the gamma distribution, with the parameters α and β, is of a high confidence with the empirical distribution.

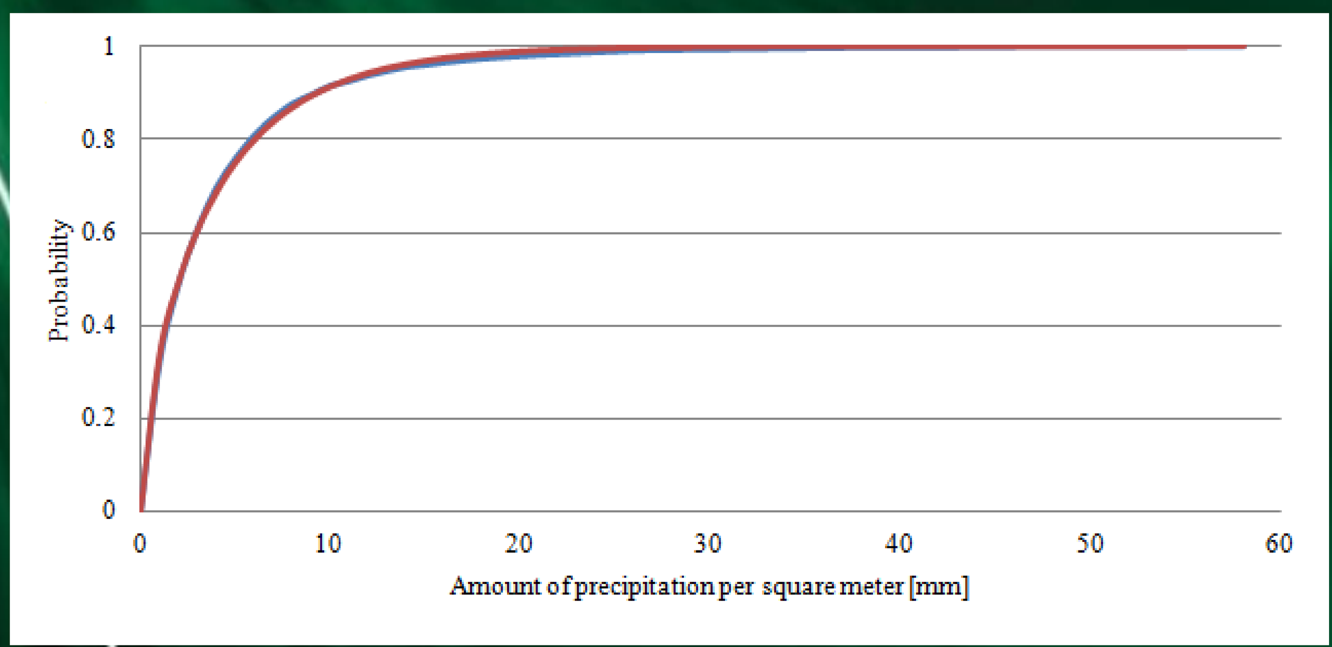

Fig. 1 Gamma distribution function and empirical distribution function for precipitation

The plot of distribution function shown in Figure 1 confirms the confidence of both the gamma and the empirical distribution.

For the length of the series without precipitation:

Based on N = 16434 days of observations of atmospheric precipitation, there were observed n = 1868 series of days without rain. The random variable tested was the length of the series. The values of basic statistics are given below.

- Number of series without rain m = 1868
- Average value = 4.308
- Standard deviation = 4.410
- Min = 1
- Max = 40
- Median = 2.322

Gamma distribution parameters determined by the most reliable method:

α = 0.9541 β =4.5150

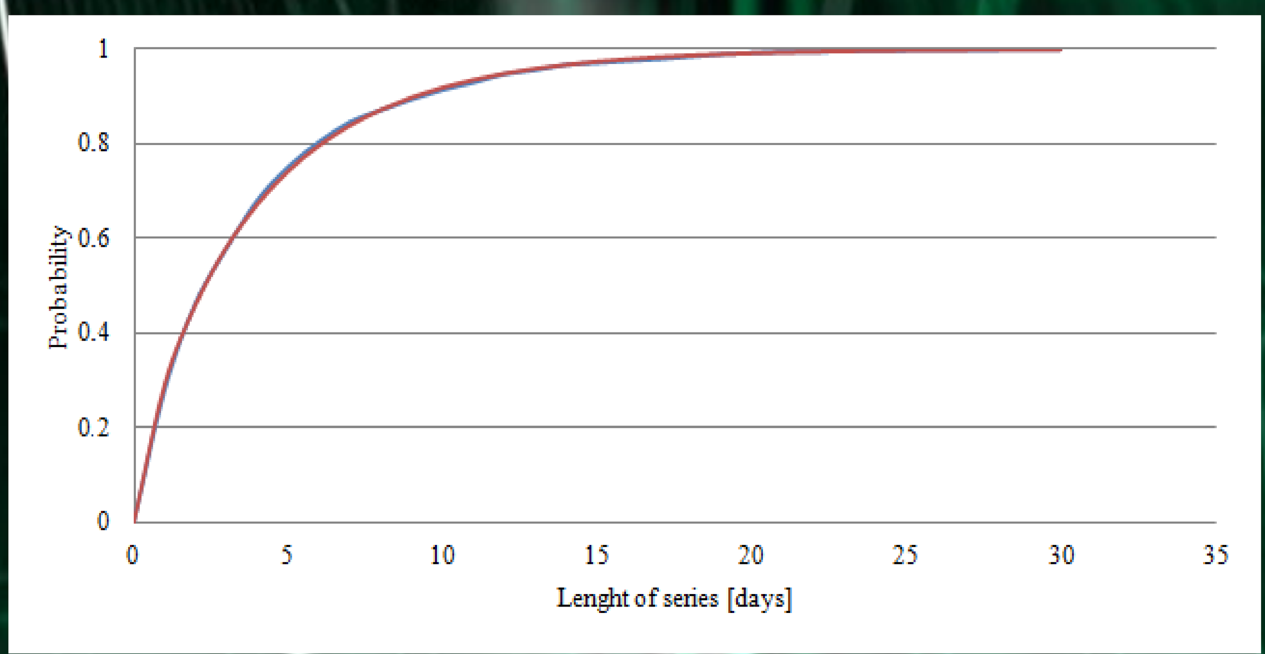

Fig. 2. Graph of empirical and gamma distribution functions

Two tests, the Kolmogorov and the Pearson χ2, were used to investigate the compatibility of the length of the series distribution with gamma distribution. It was found that the gamma distribution with the parameters α and β is consistent with the empirical distribution of the length of the series.

Both tests proved high confidence between empirical and gamma distributions. Goodness of fit with the gamma distribution is confirmed by Figures 2 and 3.

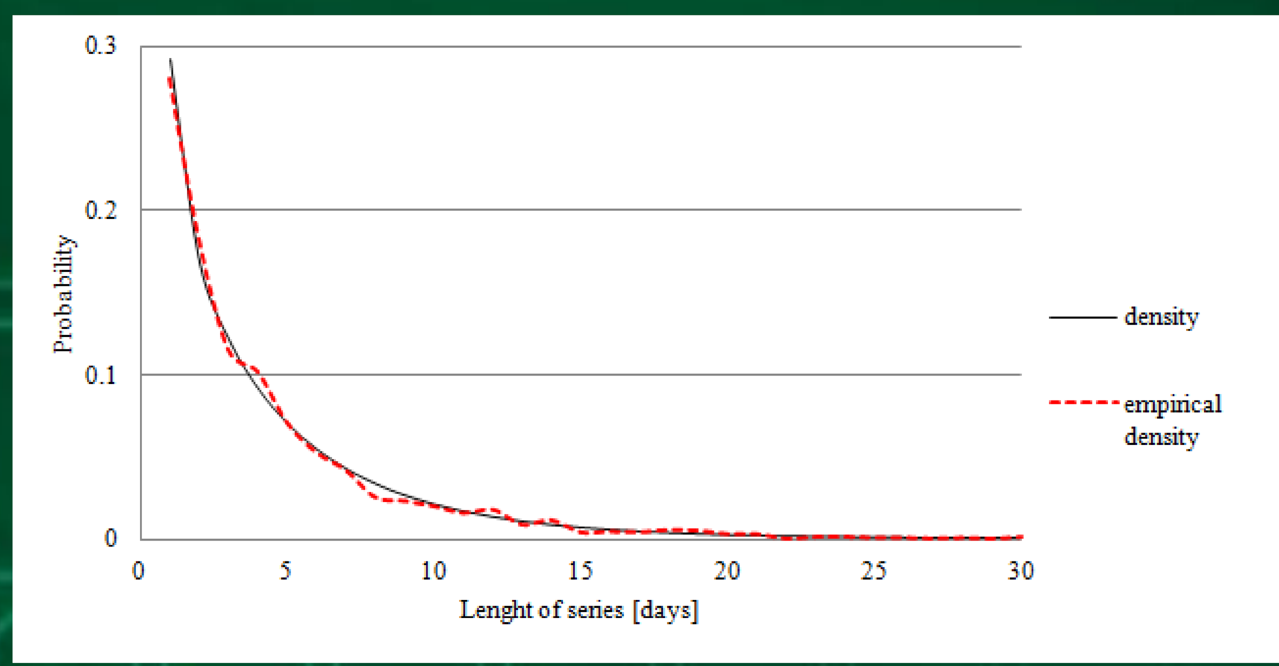

Fig. 3. Graph of the empirical density function and gamma distribution density function

By $T_{(n)}$ denotes the length of the longest series in the n-element sample. From the asymptotic theory of order statistics it is known that the random variable $T_{(n)}$ with appropriate normalization has asymptotic double exponential distribution. In particular, the limit theorem for $T_{(n)}$ can be formulated as follows: there are sequences a_n and b_n such that

$$\lim_{n\to\infty} P\{(T_{(n)} - a_n)/b_n < x\} = \exp(-\exp(-x))$$

where

$$a_n = \beta, b_n = \beta\ln(n) - \beta(\alpha-1)\ln(\beta) - \beta\ln(\Gamma(\alpha)) + \beta(\alpha-1)\ln(\beta\ln(n))$$

The limit distribution (1) makes it possible to calculate the approximate probabilities that the length of the longest series of days without rainfall does not exceed the set value. Figure 4 shows the dependence of the probability of not exceeding a given length of series.

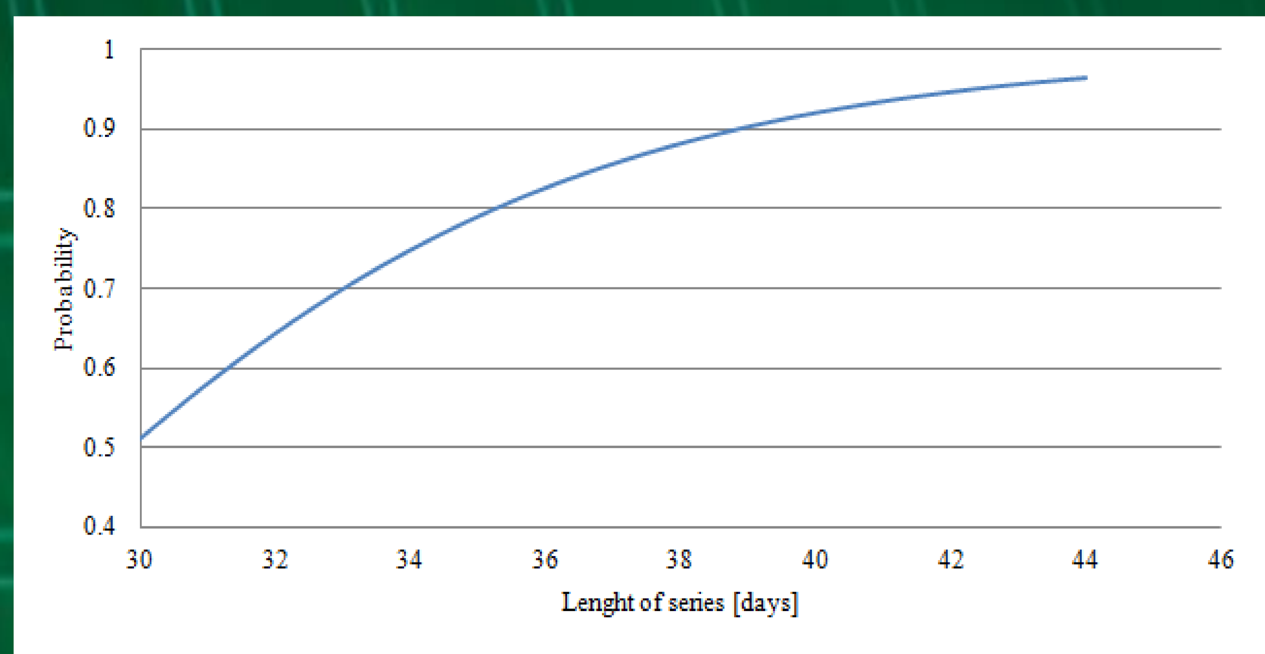

Fig. 4. The probability of not exceeding the predetermined length by the longest series

For example, the probability that the longest series length does not exceed 30 days equals to 0.49, while for 40 days the probability is 0.08. This means that in practice the occurrence of long periods without rainfall is real. It has to do with droughts occurring quite frequently in our climatic zone especially in Kuyavian & Pomeranian Province).

Conclusion

Determined probability of occurrence of the length of series of days without precipitation can be used in forecasting productive and economic effects in agriculture through application of met parameters in biophysical and economic models.