The State-contingent Approach to Production and Choice under Uncertainty: Usefulness as a Basis for Economic Modeling

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Abstract—The state-contingent approach developed by Chambers and Quiggin (2000) constitutes an attractive blend of a theory of production analysis under uncertainty and a theory of decision-making under uncertainty.

One of the goals of this contribution is to introduce the reader to the approach by outlining its contents while comparing and contrasting it to related theories. With respect to production analysis: an emphasis is made on the ability of the approach to deliver well defined cost functions corresponding to stochastic production technologies. With respect to decision-making under uncertainty: the comparison with other theories consistent with a rational agent emphasizes the production theoretical basis of the state-contingent approach.

It is the author's belief that appropriately categorizing the state-contingent approach serves the primary goal of this work: to explore its usefulness as a basis for economic modeling. Some challenges regarding an empirical implementation are discussed: challenges in estimating the parameters of a state-contingent technology representation in general, as well as challenges arising from the fact that the approach is constructed around the argument pioneered by Leonard J Savage: that probabilities underlying economic decision-making are inherently subjective.

Index Terms— decision-making under uncertainty, economic modeling, production analysis under uncertainty, state-contingent approach.

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1 The static model

The state-contingent approach involves describing the uncertain future as production outcomes y_s assigned to a finite number of mutually exclusive states of nature s (s belonging to the space of states of nature Ω). The state of nature s is perceived by an optimizing agent as occurring with probability π_s. The agent adjusts her efforts in order to ex ante maximize her utility given certain technological and cost conditions. It can be thereby argued, that apart from being capable of accommodating individual decision-making under uncertainty, the state-contingent approach overcomes basic limitations to other approaches to production analysis in an uncertain environment by elegantly extending the formulation of production technology to a correspondence between inputs and potential outputs.

An inability to adapt to uncertainty is accounted for as the extreme case of an output cubical technology, a technology which would not allow for a substitution of state-contingent outputs by rearranging inputs ex ante as illustrated on the right hand side in Fig. 1. It can be argued that a stochastic production function formulation would sufficiently account for uncertainty if the agent is faced with an output cubical technology. A state-contingent formulation would still be formally correct in this case.

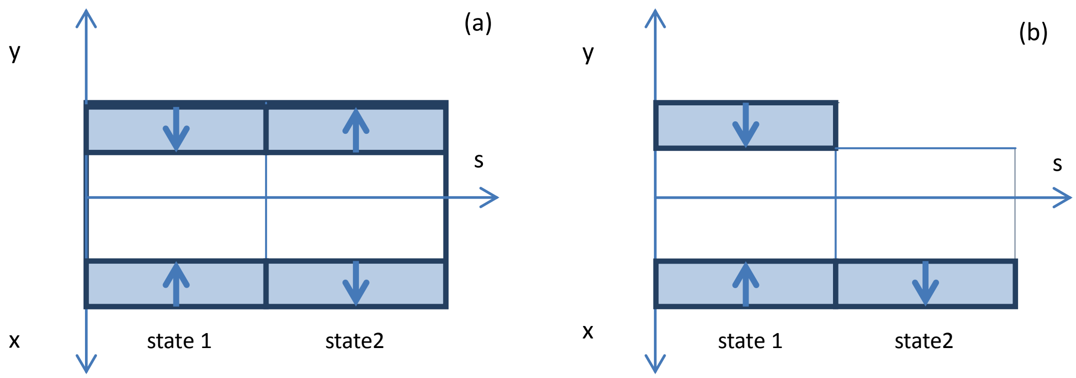

Fig. 1 Horizontal axis: states of nature, two in this example. Vertical axis: above origin aggregated agricultural output (y), below origin aggregated agricultural input (x). **(a)** General case: ex ante decreasing input in state one and reallocating it to state two would decrease output in state one and increase output in state two. Substitution between state-continent outputs is possible. **(b)** Output cubical case: ex ante decreasing input in state one and reallocating it to state two would decrease output in state one, but not increase output in state two. Substitution between state-continent outputs is not possible.
Source: own illustration.

A state-contingent production technology is defined as a mapping of the vector of inputs $x \in R_+^N$ onto the matrix of ex-ante state-contingent outputs $y \in R_+^{M \times S}$, where s (s = 1,..., S) is a state of nature and m (m = 1,..., M) identifies the output. The typical element of y, y_{ms}, reflects the amount of output m that could be produced in s. After Nature makes a draw from Ω only a single column of y occurs corresponding to the state of nature s: $y_s \in R_+^M$.

Since production economic studies often focus on analyzing production decisions (e.g. input choices) related to economic parameters (e.g. prices) dual representations of the production technology (e.g. cost functions) are an entity of particular interest. Herein lies a major disadvantage of the stochastic production frontier approach – deriving a cost function corresponding to the formulation would involve minimizing inputs with respect to a stochastic quantity.

The state contingent approach, on the other hand, is fully capable of delivering well defined cost functions corresponding to a stochastic production technology. The corresponding cost function is understood not as costs related to a certain output level, but rather as costs incurred in order to arrange ex ante for a pattern of production (inputs and outputs). Under linear input pricing the effort cost function is defined as

$$c(\mathrm{w},\mathrm{y}) = \min_{x}(\mathrm{wx} : \mathrm{x} \in \mathrm{X}(\mathrm{y}), \mathrm{w} \in \mathbb{R}_{+}^{N}) \tag{1}$$

x – input quantities	y – output quantities
w – input prices	

As can be seen, the effort cost function resembles a multi output cost function and involves the choice of optimal input amounts under certain technological conditions, stated above in terms of sets (x ϵ X(y)). These input amounts follow from the cost-minimization postulate and are thus independent of the agents' preferences.

Towards decision-making: the revenue-cost function C(w,r,p) involves decisions over inputs as well as outputs to achieve at least a certain level of state-contingent revenues:

$$c(\mathrm{w},\mathrm{r},\mathrm{p}) = \min_{y}(c(\mathrm{w},\mathrm{y}) : \sum_{m=1}^{M} p_{ms} y_{ms} \geq r_s, s \in \Omega) \tag{2}$$

y – output quantities	y_{ms} – quantity of output m in state s
w – input prices	p_{ms} – price of output m in state s
Ω – space of nature states	r_s – target revenue in state s

The revenue cost function incorporates how much an agent should at least produce in a given state, given output prices p_s, to obtain revenue of r_s as well as the cost minimizing way to produce the output quantities. The optimal state-contingent revenue mix, and thereby the optimal output quantities, are determined by the tangential point between the isocost curve associated with the revenue cost function and the agents' indifference curves. The production decisions in two extreme cases – the risk neutral agent and the extremely risk averse agent- are illustrated in Fig. 2.

What distinguishes the approach from the von Neumann-Morgenstern expected utility model is the reliance on subjective probabilities for decision-making. What distinguishes it from the subjective expected utility approach of Savage is the production origin of the set of alternatives an agent could have a preference over.

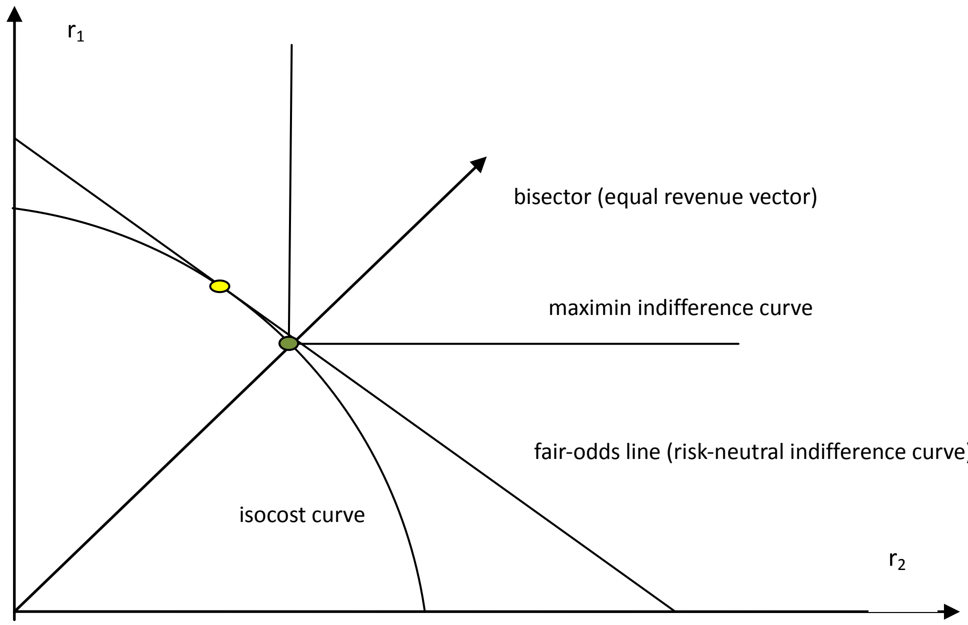

Fig. 2 The production decision with two states of nature. Yellow point for the risk neutral producer: the indifference curve of the risk neutral producer coincides with the fair-odds line, the line expressing the ratio between the subjective probabilities of state occurrence. Green point for the extremely risk-averse producer: while the extremely risk averse producer perceives the fair-odds line, her extreme preferences drive her to always choose points along the bisector where the state-contingent revenues are equal.
Source: Chambers & Quiggin (2000), page 179.

2 Challenges to empirical implementation

One of the first challenges to empirical implementation is the inherently subjective nature of the approach. The states of nature are experienced by the agents, and so are the probabilities of state occurrence. The researcher is thus faced with the costly and time consuming task of data collection in order to determine the number of states, their characteristics (in order to know when a state of nature has occurred) and the subjective probabilities attached to them, if the latter happen to be constant over time. Alternatively, the researcher can make additional assumptions regarding the agents' perceptions as the author has done in the work described below.

In general, a trade-off should be kept in mind when choosing the number of states: increasing the number of nature-states might make the set of nature-states exhaustive, but doing so leads to an issue akin to the 'curse of dimensionality' – the informational value of available observations plummets: A real world observation on production necessarily belongs to a single state of nature and would be much more informative on the parameters of a technology defined over two states of nature than on the parameters of a technology defined over ten nature-states. The researcher should be aware of the computational requirements of the estimation procedure subsequently used – since observations belong to a single state of nature one is necessarily dealing with a strongly unbalanced panel. With the choice of nature-states comes the possibility of certain states being only scarcely described by data, resulting in an inability to run even a standard panel data model.

With regard to state identification: the author has proposed a procedure for state identification based on additional assumptions regarding the agents' perceptions, which deviates from the treatment of the issue in the essential contribution of (Nauges et al. 2011). The assumption is that a researcher can infer the subjective perception of the world and its possible states by observing biophysical data: historical observations on environmental conditions and the corresponding field observations.[1] This assumption requires experimental validation.

The possible outcomes of production are by assumption being perceived by farmers in dichotomous terms. In a world with two possible crops to produce the goal of the researcher could then be to isolate three states of nature: one favorable for growing crop 1 (marked in green in Table 1), one favorable for growing crop 2 (marked in red in Table 1) and one equally good (or bad) for growing both (marked in blue in Table 1). The analysis involves putting the data from field observations in relation to one another and identifying three groups of data points by k-means clustering. Yields of farmers are subsequently attributed to one of the nature-states according to the year those experiences occurred in. The estimation of a production technology can take place and an effort-cost function can be calibrated using the parameter estimates. (Whether the dual function has a closed form representation depends on the form of the stochastic technology it corresponds to and should be checked on a case-by-case basis.)

Calibrating the revenue-cost function proves slightly trickier: while unemployment benefits or fixed costs seem like a viable representation of the target state-contingent revenues r_s, finding an estimate for output prices p_{ms} presents a challenge. Since the production decision is made ex ante it appears reasonable to assume that it is made based on output price expectations. It seems useful to develop a

[1] By field observations the author means data coming from suitable field experiments at agricultural experimental stations. These are subsequently related to the yield of farmers or agricultural yields, averaged values of agricultural output, in order to filter out the effects of environmental conditions.

function which constructs price expectations while being logically consistent with the notions used and developed in the approach.

Table 1 States of nature defined in relative terms.
Source: own illustration.

	Crop 2	
Crop 1	(good, good)	(good, bad)
	(bad, good)	(bad, bad)

Additionally the researcher has modeling freedom regarding the subjective probabilities of state occurrence which play a vital role in the decision making problem of any less than extremely risk-averse agent. A starting point consistent with the procedure outlined above would be using the relative frequency of state occurrence as suggested by field observations to represent the perspective of a backward looking agent ignoring the possibility of temporal patterns in biophysical data. Forward looking agents relying on forecasts as well as mixed strategies represent viable alternatives.

In any case, the researcher should be aware of the interplay of the form of an eventual function constructing output price expectations and the choice of subjective probabilities of state occurrence, since they govern the slopes of the isocost curve and of the indifference curve respectively. Unsubstantiated modeling choices could easily lead to arbitrary results.

3 Acknowledgements

The financial support of ScienceCampus Halle is gratefully acknowledged.

4 References

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