Application of Markov chains approach for expecting extreme precipitation changes having impact on food supply

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Plan of presentation

1. Idea of variability of weather conditions
2. Problem of risk assessement of occurence of not desirable weather conditions with Markov chains approach
3. Analysis and findings on variability of precipitation in the area of Bydgoszcz
4. Possibilities of application of obtained results in a view of continuation of undertaken research
5. Conclusion

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What is done and has to be done

- The survey was undertaken to use selected methodology, and evaluate their usefulness for forecasting yields because of changed weather conditions
- The findings are promising but should be still continued
- Narrow scope of analysis – to find out regularities in precipitation distribution in the future because of precipitation distribution in the past

Idea of variability of weather conditions

- Weather conditions such as temperature and the occurrence of frosts during the initial growing season and also precipitation are random events.
- To describe the weather as random events one can use mathematical methods of probability theory, mathematical statistics and stochastic processes.
- Randomness of weather conditions causes randomness of yields of different crops.
- Figures containing information on weather parameters and yields of various crops recorded in different years allow for the construction of mathematical models describing the dependence of the yields on weather conditions.
- In particular, previous studies [Bojar, Knopik, Żarski, 2013, Bojar et al. 2013] show an occurrence of significant relationship yields of some crops on the length of series of days without rain.

Idea of variability of weather conditions

- The basis for predicting unfavorable yields, in subsequent years, is an assessment of the probability distribution of yields in a randomly selected year.
- One method is to assess, on the basis of meteorological data, the probability distribution of the length of a series of consecutive days without rain, and on this basis, evaluation of yields.
- Attempting such an approach were presented in the papers [Bojar, Knopik, Żarski, 2013, Bojar & al. 2014].

Problem of risk assessement of occurence of not desirable weather conditions with Markov chains approach

- In this study proposes the use of a two-state Markov chains to evaluate the probability distribution of the length of the longest series of days without rain.
- The basic element of the approach to the problem of assessing the risk of adverse weather events, and related the low yields, is to evaluate the matrix of transition probabilities of the two-state Markov chain.
- The matrix has the form:

$$P = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}$$

- where p_{00} is the conditional probability that the current day will be no rain on condition that it did not rain yesterday.
- Similarly, p_{01} is the conditional probability that it rains today, provided that it did not rain yesterday.

Problem of risk assessement of occurence of not desirable weather conditions with Markov chains approach

- We know that is true equality: $p_{00} + p_{01} = 1$.
- Similarly, the probability is determined by p_{10} i p_{11} ($p_{10} + p_{11} = 1$). The evaluation of probabilities p_{00}, p_{01}, p_{10} and p_{11} is carried out based on weather data.
- These probabilities are the basis for the assessment of the probability distribution of the length of the longest series of days without rain.
- The length of days without rain series is associated with a corresponding yield.
- Weather risk in subsequent years, at least one year of "drastically" low yields is to assess the likelihood of adequately long series of days without rain in these years, e.g. for barley.

Analysis and findings on variability of precipitation in the area of Bydgoszcz

- This study uses data collected in the area of Bydgoszcz in the years 1971 – 2012.
- This data provided the basis for evaluation of transition probability matrix P of two state Markov chain X(i), where i=1,2,…,X(i) is a random process accepting two values: 0 or 1. If X(i) =0, it means that on the i - th day there was no rain, whereas if X(i) =1, on the i-th day it was raining.
- Empirical data provides the basis for assessment of conditional probabilities
- $p_{ij} = P\{ X(k+1) = i \mid X(k) = j \}$, where k = 0, 1, 2, … and i, j = 0, 1.
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- We have assumed that p_{ij} probabilities do not depend on k which means that process X(i) is homogenous.

Analysis and findings on variability of precipitation in the area of Bydgoszcz

Realization of matrix P for the years 1971 – 1980 is as follows :

$$P_1 = \begin{bmatrix} 0.49 & 0.51 \\ 0.84 & 0.16 \end{bmatrix}$$

Matrix P_2 for the years 1981-1999 was determined as:

$$P_2 = \begin{bmatrix} 0.47 & 0.53 \\ 0.83 & 0.17 \end{bmatrix}$$

Matrix P_3 for the years 1981-1999 was determined as:

$$P_3 = \begin{bmatrix} 0.43 & 0.53 \\ 0.81 & 0.19 \end{bmatrix}$$

Matrix P_4 for the years 2001-2010 was determined as:

$$P_4 = \begin{bmatrix} 0.50 & 0.50 \\ 0.88 & 0.12 \end{bmatrix}$$

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Analysis and findings on variability of precipitation in the area of Bydgoszcz

- Matrices P_1, P_2, P_3 and P_4 have been determined for the next successive years.
- An analysis of transition probability values shows that there exist certain differences between the probabilities.
- The quantity and frequency of precipitation reveal big variability.
- Estimated matrices of transition probabilities will provide the basis for investigation of a random variable equal to the length of a series of days without rain.
- The collected data concerning precipitation was used for probability assessment of the length of a series of days without rain.

Analysis and findings on variability of precipitation in the area of Bydgoszcz

- The value of the length of a series of days without rain obtained from the data allowed to calculate the values of basis statistics:
- Mean value = 3.911
- Standard deviation = 5.197
- Variability coefficient = 1.329
- Minimal value = 1.000
- Maximal value = 42.000

Analysis and findings on variability of precipitation in the area of Bydgoszcz

Next, the consistence of empirical data and gamma distribution was studied. The distribution of gamma density is expressed by a formula in the form:

$$f(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} \qquad \text{for } x > 0, \alpha > 0, \beta > 0,$$

where Γ(α) denotes gamma function defined by the formula:

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$$

Analysis and findings on variability of precipitation in the area of Bydgoszcz

The values of distribution parameters were estimated by the method maximum likelihood.: for gamma distribution were calculated $\alpha = 0.655$, $\beta = 5.722$.

- Empirical distribution function and distribution function of gamma distribution are presented in figure 1.
- The calculated value of Kolmogorov statistics λ – for the test of consistency with gamma distribution $\lambda = 0.199$.
- The calculated value of λ statistics is distinctly lower than the critical value $\lambda_p = 1.36$, for significance level $p = 0.05$.
- The calculated value of statistics indicates no grounds for rejection of the hypothesis that the analyzed data is consistent with gamma distribution.
- A plots depicted in figure 1 shows that gamma distribution is a good model for the analyzed empirical distribution.

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Analysis and findings on variability of precipitation in the area of Bydgoszcz

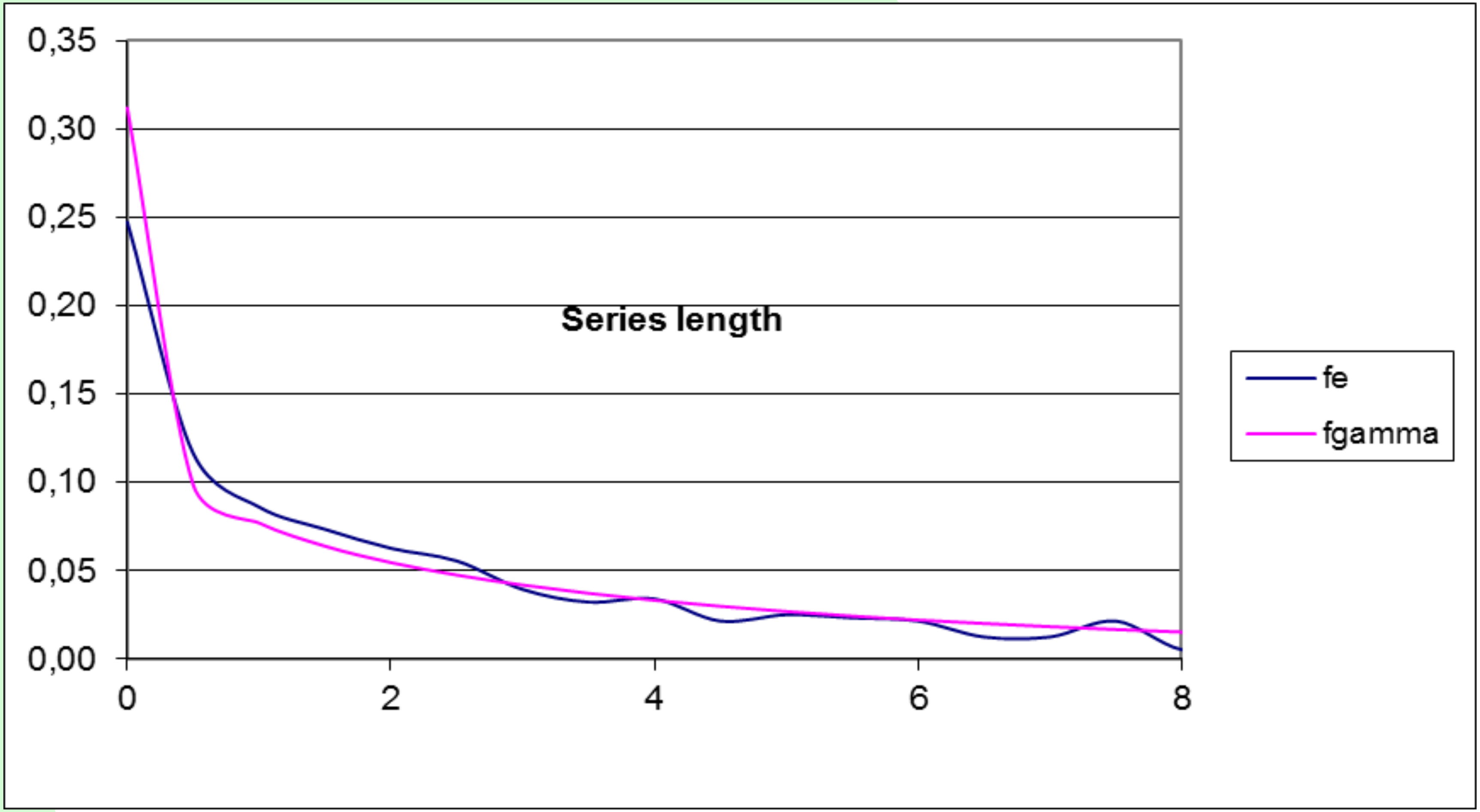

Fig. 1. Plots of empirical density and gamma distribution

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Analysis and findings on variability of precipitation in the area of Bydgoszcz

- According to this research the probability that a randomly selected day with no rain equals $p = 0.694$.
- This is a relatively high probability.
- In practice this fact justifies occurrence of a relatively long series of days without rain.
- Statistical analysis of the precipitation value leads to gamma distribution as a distribution adequate to the rainfall quantity empirical distribution.
- Estimated values of gamma distribution: $\alpha = 0.646$, $\beta = 7.371$.

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Analysis and findings on variability of precipitation in the area of Bydgoszcz

•The calculated value of Kolmogorov statistics λ – for the test of consistency with gamma distribution $\lambda = 0.303$.

•The calculated value of statistics for empirical distribution consistency with gamma distribution reveals fairly good consistence of the rainfall quantity with gamma distribution.

•The plots of the density distribution functions is shown in figure 2.

•The found dependencies between precipitation and crop yield of selected cereals will be used in regional case study analysis.

•They will be applied together with the data from CAPRI large-scale model to forecast expected changes in food supply in Kujavian Pomeranian and Lubelskie regions (Britz and Witzke 2012), (Stocco at al. 2013), (Köchy and Zimmermann. 2013).

Analysis and findings on variability of precipitation in the area of Bydgoszcz

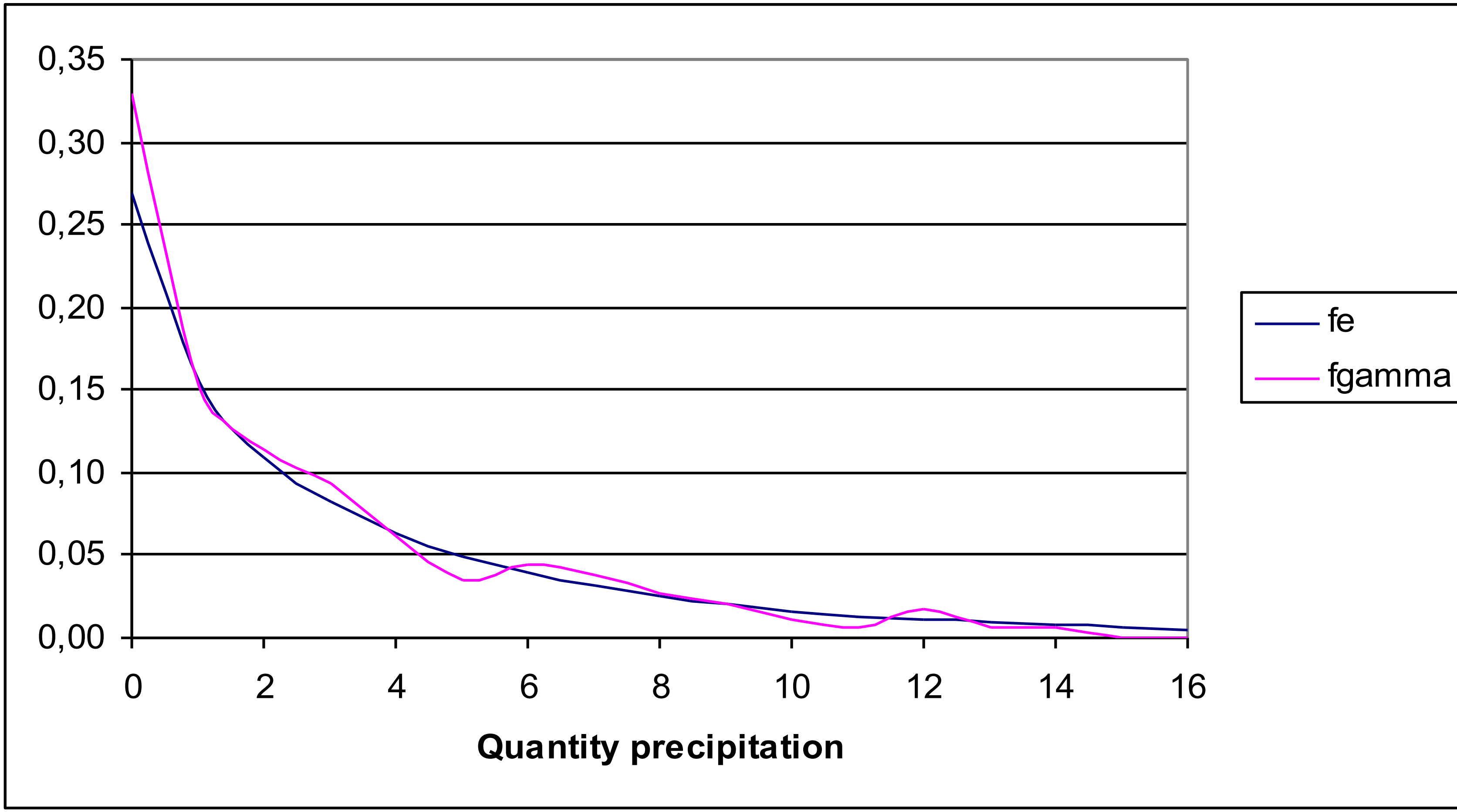

Fig. 2. Plots of empirical density and gamma distribution

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Conclusion

- This work presents a proposal of Markov two-state chain to be used as a model of change in occurrence of daily precipitation.
- Transition probability matrices of the proposed Markov chain have been evaluated on the basis of meteorological data from the years 1971-2012.
- The evaluated matrices reveal significant variability of precipitation in the considered period.
- Meteorological data from 1971 – 2012 provided the basis for assessment of probability distribution of the length of a series of days without precipitation.
- Distribution of the length of a series of days without rain is consistent with gamma distribution.

Conclusion

- Analysis of days with precipitation has revealed that the quantity of daily precipitation is also of gamma distribution.
- The findings presented in conclusions 1, 2, 3 will provide the basis for investigation of models for forecasting occurrence of undesired phenomena connected with precipitation and their relations with the expected crop yield drop of some cereals.
- The found regularities concerning variability of precipitation will be used for determination of food supply changes in long-term prognosis to come from general circulation model (precipitation, temperature) to bio-physical model (yields) and economic model (production)

THANK YOU FOR YOUR ATTENTION

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