A prototype dynamic stochastic equilibrium model of the global food system

Øyvind Hoveid
oyvind.hoveid@nilf.no

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Overview

- Motivation
- About Dynamic Stochastic General Equilibrium (DSGE)
- What needs to be modeled with regard to food security?
- How can it be done? (technical)
- How can policies support food security?

Motivation

- Need for food is relatively smooth, while supply is partly stochastic and seasonal. Food security problems $\leftarrow$ price spikes $\rightarrow$ social unrest
- For each price jump there is a cry of "wolf". *"The globe is not expected to feed all the humans anymore"*
- Price jumps so far have not been tipping points of global food balance
- New price spikes are therefore expected — and are bad enough to be analyzed and met with policies

Dynamic stochastic general equilibrium models (DSGE)

- DSGE initiated by Kydland and Prescott (1977): "Rules rather than discretion: Inconsistency of optimal plans"
- Stochastic events can and need be modeled because markets for insurance are incomplete
- Policies should follow known rules rather than some hidden agenda
- First best optimal plans do not necessarily exist with incomplete assets markets
- Models should be positive as opposed to normative

Contrast to paradigm of agricultural economics

- Equilibrium described with normative models
- Major models are static
- Stochastic events are in any case ignored with reference to Arrow-securities
- Equilibria for say 2050 are formed by changing parameters from base to 2050 levels
- Such parameter change is hazardous if not based on data and theory

Dynamic stochastic general equilibrium models (DSGE) (2)

- Fitted to data from real business cycles. Analysis of fiscal and monetary policies
- Disturbingly technical despite oversimplified models: One single consumer good is typical.
- Less elegant than normative models
- Do not even think of making the DSGE perfect!
- Bayesian estimation of variables and parameters is possible though (Smets and Wouters, 2003)
- Why not agriculture and food security?

Food security needs modeling of stochastic and seasonal supply and deterministic demand

- A number of locations deliver crops to local market at various times
- Local price is determined by local supply and global demand for consumption, processing and storage
- Heterogeneity of locations can mirror the overall heterogeneity in the world
- Production need be planned ex ante with regard to expected weather and prices at harvesting time. Ex post harvest is affected by stochastic weather at location. Ex post price is also affected by stochastic events elsewhere
- Key question: Are the incentives for storage sufficiently large to carry the population through several bad crop years in a row?

Are known crop models useful for DSGE?

- Crop models are recursive in small steps, $s = 0, \ldots, S$. k_s, g_s and n_s are state, management and nature at s

$$k_s = K_s(k_{s-1}, g_{s-1}, n_s)$$

- Management dependent crop model:

$$k_S = K\left(\{(k_{s-1}, g_{s-1}, n_s)\}_{s=1}^{S}\right)$$

- Management model. The first management action, g_0, is planting, ie. crop and cultivar decision:

$$g_s = G\left(k_s, \Pr(\{n_{s'}\}_{s'=s+1}^{S}), \Pr(p_S)\right)$$

- General ex post crop model:

$$k_S = \mathcal{K}\left(k_0, \{n_{s'}\}_{s'=1}^{S}, \Pr(\{n_{s'}\}_{s'=1}^{S}), \Pr(p_S)\right)$$

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A simplified two-period model?

- Management dependent crop model:

$$k_1 = K\left(k_0, g_0, n_1\right)$$

- Management model:

$$g_0 = G\left(k_0, \Pr(n_1, p_1)\right)$$

- Simplified ex post crop model:

$$k_1 = \mathcal{K}\left(k_0, n_1, \Pr(n_1, p_1)\right)$$

How to model dynamics?

Consider a risk averse farm household who maximize expected present utility with respect to input to household production d_t, and process management g_t, and with regard to nature n_t, prices p_t, and discounting, β:

$$\mathrm{E}_{\pi(\{n_t,p_t\}_{t=1}^{T})}\left[\sum_{t=0}^{T}\beta^t U(d_t, g_t)\right]$$

subject to a budget constraint in terms of units of account l_t and net demand from markets m_t:

$$l_{t+1} = l_t - p_t^T m_t$$

and a production model in terms of real assets k_t:

$$k_{t+1} = K(k_t + m_t - d_t, g_t, n_{t+1})$$

How to model dynamics (2)?

The maximization lead to a value function, $V_0(k, l, p)$, specifying the best outcome starting at time 0 with assets, k, l and prices p. That value function is related to $V_1(k, l, p)$, through the a Bellmann equation:

$$V_0(k, l, p) = \max_{k+m-d \geq 0} \Big[U(d, g) + \beta \mathrm{E}_{\pi(n_1, p_1)} V_1(K(k + m - d, g, n_1), l - p^T m, p_1) \Big]$$

Optimality conditions

First order conditions wrt. net demand m, household inputs d and managements g (ignoring positivity constraints)

$$\mathrm{E}\left[\partial_k V_1 \partial_k K - \partial_l V_1 p\right] = 0$$

$$\partial_d U - \beta \mathrm{E} \partial_k V_1 \partial_k K = 0$$

$$\partial_g U + \beta \mathrm{E} \partial_k V_1 \partial_g K = 0$$

Euler equation wrt. stocks (k, l) and prices p

$$\partial_k V_0 = \beta \mathrm{E}\left[\partial_k V_1 \partial_k K\right]$$

$$\partial_l V_0 = \beta \mathrm{E}\left[\partial_l V_1\right]$$

$$\partial_p V_0 = -\beta \mathrm{E}\left[\partial_l V_1 m\right]$$

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How to deal with expectations in optimality conditions?

Quadratic approximation: For any function $F(x)$ with x stochastic

$$
\begin{aligned}
&\mathrm{E}F(x)\\
&\approx \mathrm{E}\left[F(\mathrm{E}x) + \partial F(\mathrm{E}x)(x-\mathrm{E}x) + \frac{1}{2}(x-\mathrm{E}x)^T\partial^2 F(\mathrm{E}x)(x-\mathrm{E}x)\right]\\
&= F(\mathrm{E}x) + \frac{1}{2}\mathrm{trace}[\partial^2 F(\mathrm{E}x)\mathrm{Var}\ x]
\end{aligned}
$$

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Model closure

- Consumers are like producer households with similar optimality conditions except their k is fixed and $g = 0$
- The excess net demand m_i from all agents i in the economy:

$$\sum_i m_i = 0$$

- Optimal values $x = (\{d_i, g_i, k_i, l_i, m_i\}_i, p)$ are approximately consistent with individual optimal planning and clearing of spot markets
- But individual plans are not coordinated, and are not the result of a joint maximization of all agents expected present utility

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How to deal with functions and probability distributions?

- Consider value functions parametric with agent and time specific parameters γ_t^t for the ex post functions like V_0 and γ_t^{t-1} for the ex ante functions like V_1.
- Under strong assumptions they will all be identical. If models are misspecified they are probably not.
- Consider probability distributions, utility functions and process functions parametric with parameters γ_t^π, γ_t^U and γ_t^K.
- Optimality conditions, $G(x, \gamma) = 0$, define endogenous variables $x = (d, g, k, l, m, p)$ implicitly as non-linear functions of parameters γ

$$x = X(\gamma)$$

Semi-Bayesian estimation

- Subset of variables are observed with normal error, $\epsilon = y - \overline{X}(\gamma) \sim \mathcal{N}(0, \sigma)$.
- Parameters γ are multivariate normal (after transformation), $\gamma = \mathcal{N}(0, \Sigma)$.
- Find σ, Σ to maximize likelihood

$$\pi(y|\sigma, \Sigma) = \int \phi(y - \overline{X}(\gamma); \sigma)\phi(\gamma; \Sigma)d\gamma$$

Semi-Bayesian estimation (2)

- Find mode of integrand, $\gamma^* = \gamma^*(\sigma, \Sigma)$:

$$\partial_\gamma[\phi(y - \overline{X}(\gamma^*); \sigma)\phi(\gamma^*; \Sigma)] = 0$$

- Approximate $\overline{X}(\gamma)$ linearly at γ^*:

$$\overline{X}(\gamma) \approx \overline{X}(\gamma^*) + \partial_\gamma \overline{X}(\gamma^*)(\gamma - \gamma^*) = D_0 + D_1(\gamma - \gamma^*)$$

- Using approximation it can be shown that:

$$\pi(y|\Sigma, \sigma) \approx \phi(y - D_0; \sigma + D_1 \Sigma D_1^T)$$

- With careful maximization avoiding $\sigma = 0$ and $\Sigma = 0$, appropriate values can be found.

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Semi-Bayesian estimation (3)

- If more modes than one, say $(\gamma_1^*, \ldots, \gamma_K^*)$, a kernel density arise:

$$\pi(y|\Sigma,\sigma) \approx \sum_k w_k \phi(y - D_{k0};\ \left(\sigma + D_{k1}\Sigma D_{k1}^T\right))$$

- Weights $w_k \propto \phi(y - \overline{X}(\gamma_k^*); \sigma)\phi(\gamma_k^*; \Sigma)$

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Why all these technicalities?

- Needed to find a dynamic stochastic mechanism which is approximately consistent with individual optimal planning under uncertainty, market clearing of spot markets, and observations thereof
- Then we may ask:
- What are the probabilities of certain extreme events w.r.t. price spikes?
- What are the improvements of producer and consumers present expected utility when some new policy mechanism is introduced?
- What can be predicted for near future?
- What are likely stories to be told?

What are likely stories to be told?

- Say producers manage storage and there is a bad crop year — by chance. Price run high and stocks low. Next year is also a bad year. Price run even higher and stocks even lower. Third year follows the same way. Food balance can be a disaster. What can be learned?
- Adaptive expectations of producers would have eased the situation for consumers, but is beyond control.
- Price control would be popular first year, but would empty stocks faster, and cause more problems in second and third year
- If only consumers matter, emptying of stocks in bad crop years should be avoided. There is always a chance that next year will be bad, and a given stock should be reserved for this. This requires a food security policy of storage, though.

Thanks for the attention